***CALCULATING A DERIVATIVE***

***Derivative from sum, product and quotient***

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| If each of the functions $u$ and $v$ has a derivative in point $x$,then the $Cu (C=const)$ и sum, difference, product и quotient of the functions $u$ and $v$ (in the case of a quotient it should be assumed that $v(x)\ne 0$), they also have a derivative in the point $x$ where the formulas apply:$$\left[Cu\left(x\right)\right]^{'}=Cu^{'}\left(x\right), C=const$$$$\left[u\left(x\right)\pm v\left(x\right)\right]^{'}=u^{'}\left(x\right)\pm v^{'}\left(x\right)$$$$\left[u\left(x\right)∙v\left(x\right)\right]^{'}=u^{'}\left(x\right)∙v\left(x\right)+u\left(x\right)∙v^{'}\left(x\right)$$$$\left[\frac{u\left(x\right)}{v\left(x\right)}\right]^{'}=\frac{u^{'}\left(x\right)∙v\left(x\right)-u\left(x\right)∙v^{'}\left(x\right)}{v^{2}\left(x\right)}$$ |

***Example1.*** СBy applying the rules mentioned above, we are now left with the possibility to calculate the derivative without any effort of any polynomial.. Using the following:$\left(x^{n}\right)^{'}=nx^{n-1}, n\in N$

For example, for the following polynomial we have:

$P(x)=2x^{5}+4x^{4}-3x+2$

$P^{'}(x)=\left(2x^{5}+4x^{4}-3x+2\right)^{'}$

$P^{'}(x)=\left(2x^{5}\right)^{'}+\left(4x^{4}\right)^{'}-\left(3x\right)^{'}+\left(2\right)^{'}$

$P^{'}(x)=2\left(x^{5}\right)^{'}+4\left(x^{4}\right)^{'}-3\left(x\right)^{'}+0$

$P^{'}(x)=2∙5x^{4}+4∙4x^{3}-3∙1$

$P^{'}(x)=10x^{4}+16x^{3}-3$

***Example2.*** By applying the rules mentioned above and the derivatives of some elementary functions, for the derivative of the$y=e^{x}sinx$ *we have*:

$\left(e^{x}\right)^{'}=e^{x} , \left(sinx\right)^{'}=cosx$

$$y^{'}=\left(e^{x}sinx\right)^{'}=\left(e^{x}\right)^{'}sinx+e^{'}\left(sinx\right)^{'}=e^{x}sinx+e^{x}cos=e^{x}\left(sinx+cosx\right)$$

***Example3.*** *By applying the rule for a derivative from a* quotient, for the derivative of the function$y=\frac{lnx}{x^{2}}$we have:

$$y^{'}=\left(\frac{lnx}{x^{2}}\right)^{'}=\frac{\left(lnx\right)^{'}∙x^{2}-lnx∙\left(x^{2}\right)^{'}}{\left(x^{2}\right)^{2}}=\frac{\frac{1}{x}∙x^{2}-lnx∙2x}{x^{4}}=$$

$$=\frac{x-2xlnx}{x^{4}}=\frac{x\left(1-2lnx\right)}{x^{4}}==\frac{1-2lnx}{x^{3}}$$

***Derivative from a complex function***

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| If the function $u$ has a derivative in a fixed pont $x$ , and function $y=f(u)$ has a derivative in the point $u=u(x)$ , then the function $f=f\left(u(x)\right)$ has a derivative in the point $x$ while the following formula applies $y^{'}=f^{'}(u)∙u^{'}(x)$ $\left[the formula is often written in the following form y\_{x}^{'}=f\_{u}^{'}∙u\_{x}^{'}\right]$ |

***Example4.*** In the case of the function $y=e^{sinx}$, by adding the $u=sinx$ , we get that $y^{'}=\left(e^{u}\right)\_{u}^{'}\left(sinx\right)\_{x}^{'}=e^{u}cosx=e^{sinx}cosx$

Similar for the function $=ln\left(3x^{2}-1\right)$ , by adding $u=3x^{2}-1$, we get that $y^{'}=\left(lnu\right)\_{u}^{'}∙\left(3x^{2}-1\right)\_{x}^{'}=\frac{1}{u}∙6x=\frac{6x}{3x^{2}-1}$

***Example 5.*** Calculate the derivative of the function $y=x^{x} \left(x>0\right)$

The given function we can write it in the following form $y=e^{xlnx}$ , or $=e^{u}$ , where $u=xlnx$ and we get:

 $y^{'}=\left(e^{u}\right)\_{x}^{'}=e^{u}\left(xlnx\right)^{'}=e^{xlnx}\left(1∙lnx+x∙\frac{1}{x}\right)=e^{xlnx}\left(lnx+1\right)$

***Derivative from a inverse function***

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| For the function $y=f(x)$ exists an inverse function $x=f^{-1}(y)$ in the surroundings of the point $x\_{0}$. If we have an existing derivative of the function $y=f(x)$ in point $x\_{0}$ and while $f^{'}(x\_{0})\ne 0$ ,then a derivative exists of the inverse function $x=f^{-1}(y)$ in the point $y\_{0}=f\left(x\_{0}\right)$ and is equal to $\frac{1}{f^{-1}(x\_{0})}$ . |

***Example6.*** The function $y=log\_{a}x$ is inverse to the function $=a^{y}$ $\left(where 0<a\ne 1 и x>0\right)$.

$y^{'}=\left(log\_{a}x\right)^{'}=\frac{1}{\left(a^{y}\right)^{'}}=\frac{1}{a^{y}lna}=\frac{1}{xlna}$ $\left(0<a\ne 1 ,x>0\right)$

***Example7.*** Function $y=arcsinx$ is inverse to the function $=siny$ , $\left(where -\frac{π}{2}\leq y\leq \frac{π}{2} \right)$.

$y^{'}=\left(arcsinx\right)^{'}=\frac{1}{(siny)^{'}}=\frac{1}{cosy}=\frac{1}{\sqrt{1-sin^{2}y}}=\frac{1}{\sqrt{1-x^{2}}}$ $\left(-1<x<1\right)$

In the points $x=-1$ и $x=1$ we can only discuss for a left, or right derivative of this function.

***Derivative from a implicitly defined function***

Lets assume that the values of two variables $x$ and $y$ are tied with the equation, $\left(x,y\right)=0$ .

If the function $y=f(x)$ defined on a interval $\left(a,b\right)$ is such by replacing of $y$ with $f(x)$ во $F\left(x,y\right)=0$ we get the identity of $$ , we say that $y=f(x)$ is an implicit function set with the equation $F\left(x,y\right)=0$.

***Example 8.*** We have the following function $x^{2}+y^{2}-a^{2}=0$.

If we get a derivative from both sides of the equation of $x$, assuming that $y$ is a function of $x$ , we get the following:

$2x+2yy^{'}=0$

from which it follows that

$y^{'}=-\frac{x}{y}$.

***Example9.*** We have the function $x^{6}-y-x^{2}=0$.

If we get a derivative from both sides of the equation of $x$, assuming that $y$ is a function of $x$ , we get the following:

$6y^{5}y^{'}-y^{'}-2x=0$

from which it follows that

$y^{'}=\frac{2x}{6y^{5}-1}$.

***Derivative from a parametrically defined function***

We have the following equations

$x=φ\left(t\right) , y=ω(t)$ (1)

Where $t$ takes values in the $\left[T\_{1},T\_{2}\right]$. For every value of $t$ the values $x$ и $y$ comply . If the obtained values $x$ и $y$ we interpret tem as coordinates in the coordinate plane $xOy$ , then for every value of $t$ corresponds a point in the plane. By that way, when $t$ changes from $T\_{1}$ to $T\_{2}$ , we get a curve in the plane. The equations(1) are called paсе нарекуваат parametric equations of that curve, $t$ is called a parameter, and the way of setting the curve is called а parametric.

If we assume that the function $x=φ(t)$ has an inverse function, $t=Ф(x)$,

then $y$ is a function of $x$ , or

$y=ω\left(Ф(x)\right)$ *.* (2)

That way the equations (1) define a function $y$ from $x$ , for which we say that its a given in a parameter way. The direct dependence $y=f(x)$ is obtained by eliminating the parameter $t$ from equations (1).

Lets find a derivative from the function $y$ of $x$ given in a parameter way with the equations (1). We will assume that the functions $x=φ(t)$ and $y=ω(t)$ have a derivative in each inner point of the segment $\left[T\_{1},T\_{2}\right]$ , while the function $x=φ(t)$ has an inverse $t=Ф(x)$ of the segment at hand. Then the function $y=f(x)$ defined with parameter equations (1) can be looked as a complex function.

$$y=ω\left(t\right), t=Ф(x)$$

where $t$ changes in the segmentс$\left[T\_{1},T\_{2}\right]$. Following the rule for a derivative from a complex function we find that:

$ y\_{x}^{'}=y\_{t}^{'}t\_{x}^{'}=ω\_{t}^{'}\left(t\right)Ф\_{x}^{'}(x)$ .

Based on the derivative of the inverse function we have $Ф\_{x}^{'}\left(x\right)=\frac{1}{φ\_{t}^{'}(t)}$.

By the last equation we get

$y\_{x}^{'}=\frac{ω\_{t}^{'}(t)}{φ\_{t}^{'}(t)}$ , or $y\_{x}^{'}=\frac{y\_{t}^{'}}{x\_{t}^{'}}$.

***Example10.*** Function $y$ from $x$ is given with the parametric equations:

$x=acost$

$y=asint$ $0\leq t\leq π$ .

For the arbitrary value of parameter произволна $t$ of the given segment we have the following:

$ y\_{x}^{'}=\frac{\left(asint\right)^{'}}{\left(acost\right)^{'}}=\frac{acost}{-asint}=-ctgt$ .

***TASKS***

Find the derivative of the functions, if:

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**2.  **

**3.  **

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**10. **

**11.  **

**12.  **